Chapter 1 – Reviewing linear equations

Solutions to Exercise 1A

1 a $x + 3 = 6$

$\therefore \quad x = 3$

b $x - 3 = 6$

$\therefore \quad x = 9$

c $3 - x = 2$

$-x = -1$

$\therefore \quad x = 1$

d $x + 6 = -2$

$x + 8 = 0$

$\therefore \quad x = -8$

e $2 - x = -3$

$-x = -5$

$\therefore \quad x = 5$

f $2x = 4$

$\therefore \quad x = 2$

g $3x = 5$

$\therefore \quad x = \frac{5}{3}$

h $-2x = 7$

$\therefore \quad x = -\frac{7}{2}$

i $-3x = -7$

$\therefore \quad x = \frac{7}{3}$

j $\frac{3x}{4} = 5$

$3x = 20$

$\therefore \quad x = \frac{20}{3}$

k $-\frac{3x}{5} = 2$

$-3x = 10$

$\therefore \quad x = -\frac{10}{3}$

l $-\frac{5x}{7} = -2$

$-5x = -14$

$\therefore \quad x = \frac{-14}{-5} = \frac{14}{5}$

2 a $x - b = a$

$\therefore \quad x = a + b$

b $x + b = a$

$\therefore \quad x = a - b$

c $ax = b$

$\therefore \quad x = \frac{b}{a}$

d $\frac{x}{a} = b$

$\therefore \quad x = ab$

e $\frac{ax}{b} = c$

$ax = bc$

$\therefore \quad x = \frac{bc}{a}$

3 **a**

$$2y - 4 = 6$$
$$2y = 10$$
$$y = 5$$

b

$$3t + 2 = 17$$
$$3t = 15$$
$$t = 5$$

c

$$2y + 5 = 2$$
$$2y = -3$$
$$y = -\frac{3}{2}$$

d

$$7x - 9 = 5$$
$$7x = 14$$
$$x = 2$$

e

$$2a - 4 = 7$$
$$2a = 11$$
$$a = \frac{11}{2}$$

f

$$3a + 6 = 14$$
$$3a = 8$$
$$a = \frac{8}{3}$$

g

$$\frac{y}{8} - 11 = 6$$
$$\frac{y}{8} = 17$$
$$y = 136$$

h

$$\frac{t}{3} + \frac{1}{6} = \frac{1}{2}$$
$$\frac{t}{3} = \frac{1}{3}$$
$$t = 1$$

i

$$\frac{x}{3} + 5 = 9$$
$$\frac{x}{3} = 4$$
$$x = 12$$

j

$$3 - 5y = 12$$
$$-5y = 9$$
$$y = -\frac{9}{5}$$

k

$$-3x - 7 = 14$$
$$-3x = 21$$
$$x = -7$$

l

$$14 - 3y = 8$$
$$-3y = -6$$
$$y = 2$$

4 **a** $6x - 4 = 3x$

$$3x = 4$$
$$\therefore \quad x = \frac{4}{3}$$

b $x - 5 = 4x + 10$

$$-3x = 15$$
$$\therefore \quad x = \frac{15}{-3} = -5$$

c $3x - 2 = 8 - 2x$

$5x = 10$

$\therefore \quad x = 2$

5 a $2(y + 6) = 10$

$y + 6 = 5$

$\therefore \quad y = 5 - 6 = -1$

b $2y + 6 = 3(y - 4)$

$2y + 6 = 3y - 12$

$-y = -18$

$\therefore \quad y = 18$

c $2(x + 4) = 7x + 2$

$2x + 8 = 7x + 2$

$-5x = -6$

$\therefore \quad x = \frac{6}{5}$

d $5(y - 3) = 2(2y + 4)$

$5y - 15 = 4y + 8$

$5y - 4y = 18 + 8$

$\therefore \quad y = 23$

e $x - 6 = 2(x - 3)$

$x - 6 = 2x - 6$

$-x = 0$

$\therefore \quad x = 0$

f $\frac{y + 2}{3} = 4$

$y + 2 = 12$

$\therefore \quad y = 10$

g $\frac{x}{2} + \frac{x}{3} = 10$

$\frac{5x}{6} = 10$

$5x = 60$

$\therefore \quad x = 12$

h $x + 4 = \frac{3x}{2}$

$-\frac{x}{2} = -4$

$-x = -8$

$\therefore \quad x = 8$

i $\frac{7x + 3}{2} = \frac{9x - 8}{4}$

$14x + 6 = 9x - 8$

$5x = -14$

$\therefore \quad x = -\frac{14}{5}$

j $\frac{2}{3}(1 - 2x) - 2x = -\frac{2}{5} + \frac{4}{3}(2 - 3x)$

$10(1 - 2x) - 30x = -6 + 20(2 - 3x)$

$10 - 20x - 30x = -6 + 40 - 60x$

$10x = 24$

$\therefore \quad x = \frac{12}{5}$

k $\frac{4y - 5}{2} - \frac{2y - 1}{6} = y$

$(12y - 15) - (2y - 1) = 6y$

$12y - 15 - 2y + 1 = 6y$

$4y = 14$

$\therefore \quad y = \frac{7}{2}$

6 a $ax + b = 0$

$ax = -b$

$\therefore \quad x = -\dfrac{b}{a}$

b $cx + d = e$

$cx = e - d$

$\therefore \quad x = \dfrac{e - d}{c}$

c $a(x + b) = c$

$x + b = \dfrac{c}{a}$

$\therefore \quad x = \dfrac{c}{a} - b$

d $ax + b = cx$

$ax - cx = -b$

$x(c - a) = b$

$\therefore \quad x = \dfrac{b}{c - a}$

e $\dfrac{x}{a} + \dfrac{x}{b} = 1$

$bx + ax = ab$

$x(a + b) = ab$

$\therefore \quad x = \dfrac{ab}{a + b}$

f $\dfrac{a}{x} + \dfrac{b}{x} = 1$

$\therefore \quad x = a + b$

g $ax - b = cx - d$

$ax - cx = b - d$

$x(a - c) = b - d$

$\therefore \quad x = \dfrac{b - d}{a - c}$

h $\dfrac{ax + c}{b} = d$

$ax + c = bd$

$ax = bd - c$

$\therefore \quad x = \dfrac{bd - c}{a}$

7 a $0.2x + 6 = 2.4$

$0.2x = -3.6$

$\therefore \quad x = -18$

b $0.6(2.8 - x) = 48.6$

$2.8 - x = 81$

$-x = 78.2$

$\therefore \quad x = -78.2$

c $\dfrac{2x + 12}{7} = 6.5$

$2x + 12 = 45.5$

$x + 6 = 22.75$

$\therefore \quad x = 16.75$

d $0.5x - 4 = 10$

$0.5x = 14$

$\therefore \quad x = 28$

e $\dfrac{1}{4}(x - 10) = 6$

$x - 10 = 24$

$\therefore \quad x = 34$

f $6.4x + 2 = 3.2 - 4x$

$10.4x = 1.2$

$\therefore \quad x = \dfrac{1.2}{10.4} = \dfrac{3}{26}$

8

$$\frac{b-cx}{a}+\frac{a-cx}{b}+2=0$$

$$b(b-cx)+a(a-cx)+2ab=0$$

$$b^2-bcx+a^2-acx+2ab=0$$

$$b^2+a^2+2ab=acx+bcx$$

$$(a+b)^2=cx(a+b)$$

$$\therefore\quad x=\frac{a+b}{c}$$

9

$$\frac{a}{x+a}+\frac{b}{x-b}=\frac{a+b}{x+c}$$

$$\frac{a(x-b)+b(x+a)}{(x+a)(x-b)}=\frac{a+b}{x+c}$$

$$\frac{ax-ab+bx+ab}{(x+a)(x-b)}=\frac{a+b}{x+c}$$

$$\frac{ax+bx}{(x+a)(x-b)}=\frac{a+b}{x+c}$$

$$\frac{x}{(x+a)(x-b)}=\frac{1}{x+c}$$

$$x(x+c)=(x+a)(x-b)$$

$$x^2+cx=x^2+ax-bx-ab$$

$$cx-ax+bx=-ab$$

$$x(a-b-c)=ab$$

$$\therefore\quad x=\frac{ab}{a-b-c}$$

Solutions to Exercise 1B

1 **a** $x + 2 = 6$

$\therefore \quad x = 4$

b $3x = 10$

$\therefore \quad x = \frac{10}{3}$

c $3x + 6 = 22$

$3x = 16$

$\therefore \quad x = \frac{16}{3}$

d $3x - 5 = 15$

$3x = 20$

$\therefore \quad x = \frac{20}{3}$

e $6(x + 3) = 56$

$x + 3 = \frac{56}{6} = \frac{28}{3}$

$\therefore \quad x = \frac{19}{3}$

f $\frac{x + 5}{4} = 23$

$x + 5 = 92$

$\therefore \quad x = 87$

2 $A + 3A + 2A = 48$

$6A = 48$

$\therefore \quad A = 8$

A gets \$8, B \$24 and C \$16

3 $y = 2x; x + y = 42 = 3x$

$x = \frac{42}{3}$

$\therefore \quad x = 14, y = 28$

4 $\frac{x}{3} + \frac{1}{3} = 3$

$x + 1 = 9$

$\therefore \quad x = 8$ kg

5 $L = w + 0.5; A = Lw$

$P = 2(L + w)$

$= 2(2w + 0.5)$

$= 4w + 1$

$4w + 1 = 4.8$

$4w = 3.8$

$\therefore \quad w = 0.95$

$A = 0.95(0.95 + 0.5)$

$= 1.3775 \text{ m}^2$

6 $(n - 1) + n + (n + 1) = 150$

$3n = 150$

$\therefore \quad n = 50$

Sequence = 49, 50 & 51, assuming n is the middle number.

7 $n + (n + 2) + (n + 4) + (n + 6) = 80$

$4n + 12 = 80$

$4n = 68$

$\therefore \quad n = 17$

17, 19, 21 and 23 are the odd numbers.

8 $6(x - 3000) = x + 3000$

$6x - 18000 = x + 3000$

$5x = 21000$

$\therefore \quad x = 4200$ L

9 $140(p - 3) = 120p$

$140p - 420 = 120p$

$20p = 420$

$\therefore \quad p = 21$

10 $\dfrac{x}{6} + \dfrac{x}{10} = \dfrac{48}{60}$

$5x + 3x = 24$

$8x = 24$

$x = 3$ km

11 Profit = x for crate 1 and $0.5x$ for crate 2, where x = amount of dozen eggs in each crate.

$x + \dfrac{x + 3}{2} = 15$

$2x + x + 3 = 30$

$3x = 27$

$\therefore \quad x = 9$

Crate 1 has 9 dozen, crate 2 has 12 dozen.

12 $3\left(\dfrac{45}{60}\right) + x\left(\dfrac{30}{60}\right) = 6$

$\dfrac{9}{4} + \dfrac{x}{2} = 6$

$\dfrac{x}{2} = \dfrac{15}{4}$

$\therefore \quad x = \dfrac{15}{2} = 7.5$km/hr

13

$t = \dfrac{x}{4} + \dfrac{x}{6} = \dfrac{45}{60}$

$60 \times \dfrac{x}{4} + 60 \times \dfrac{x}{6} = 45$

$15x + 10x = 45$

$25x = 45$

$x = \dfrac{45}{25}$

$= \dfrac{9}{5}$

$= 1.8$

Total $= 2 \times 1.8$

$= 3.6$ km (there and back)

Total $= 4 \times 0.9$

$= 3.6$ km there and back twice

14 $f = b + 24$

$(f + 2) + (b + 2) = 40$

$b + 26 + b + 2 = 40$

$2b = 12$

$\therefore \quad b = 6$

The boy is 6, the father 30.

Solutions to Exercise 1C

1 a $y = 2x + 1 = 3x + 2$

$-x = 1, \therefore x = -1$

$\therefore y = 2(-1) + 1 = -1$

b $y = 5x - 4 = 3x + 6$

$2x = 10, \therefore x = 5$

$\therefore y = 5(5) - 4 = 21$

c $y = 2 - 3x = 5x + 10$

$-8x = 8, \therefore x = -1$

$\therefore y = 2 - 3(-1) = 5$

d $y - 4 = 3x$ (1)

$y - 5x + 6 = 0$ (2)

From (1) $y = 3x + 4$

Subsitute in (2).

$$\begin{aligned} 3x + 4 - 5x + 6 &= 0 \\ -2x + 10 &= 0 \\ x &= 5 \end{aligned}$$

Substitute in (1). $y - 4 = 15$.

Therefore $x = 5$ and $y = 19$.

e $y - 4x = 3$ (1)

$2y - 5x + 6 = 0$ (2)

From (1) $y = 4x + 3$

Subsitute in (2).

$$\begin{aligned} 2(4x + 3) - 5x + 6 &= 0 \\ 3x + 12 &= 0 \\ x &= -4 \end{aligned}$$

Substitute in (1). $y + 16 = 3$.

Therefore $x = -4$ and $y = -13$.

f $y - 4x = 6$ (1)

$2y - 3x = 4$ (2)

From (1) $y = 4x + 6$

Subsitute in (2).

$$\begin{aligned} 2(4x + 6) - 3x &= 4 \\ 5x + 12 &= 4 \\ 5x &= -8 \\ x &= -\frac{8}{5} \end{aligned}$$

Substitute in (1). $y - 4 \times \left(-\frac{8}{5}\right) = 6$.

$y = \frac{50}{3}$

Therefore $x = -\frac{8}{5}$ and $y = -\frac{2}{5}$.

2 a

$$\begin{array}{l} x + y = 6 \\ \underline{x - y = 10} \\ 2x \quad = 16 \end{array}$$

$\therefore \quad x = 8; y = 6 - 8 = -2$

b

$$\begin{array}{l} y - x = 5 \\ \underline{y + x = 3} \\ 2y \quad = 8 \end{array}$$

$\therefore \quad y = 4; x = 3 - 4 = -1$

c

$$\begin{array}{l} x - 2y = 6 \\ \underline{-(x + 6y = 10)} \\ -8y \quad = -4 \end{array}$$

$\therefore \; y = \frac{1}{2}, \; x = 6 + \frac{2}{2} = 7$

3 a

$$\begin{array}{l} 2x - 3y = 7 \\ \underline{9x + 3y = 15} \\ 11x \quad = 22 \end{array}$$

$\therefore \; x = 2$

$4 - 3y = 7, \therefore \; y = -1$

b

$$\begin{array}{l} 4x - 10y = 20 \\ \underline{-(4x + 3y = 7)} \end{array}$$

$-13y = 13$
$\therefore\ y = -1$
$4x - 3 = 7, \therefore\ x = 2.5$

c $4m - 2n = 2$
$$\begin{array}{rl} m + 2n &= 8 \\ \hline 5m &= 10 \end{array}$$
$\therefore\ m = 2$
$8 - 2n = 2, \therefore\ n = 3$

d $14x - 12y = 40$
$$\begin{array}{rl} 9x + 12y &= 6 \\ \hline 23x &= 46 \end{array}$$
$\therefore\ x = 2$
$14 - 6y = 20, \therefore y = -1$

e $6s - 2t = 2$
$$\begin{array}{rl} 5s + 2t &= 20 \\ \hline 11s &= 22 \end{array}$$
$\therefore\ s = 2$
$6 - t = 1, \therefore\ t = 5$

f $16x - 12y = 4$
$$\begin{array}{rl} -15x + 12y &= 6 \\ \hline x &= 10 \end{array}$$
$\therefore\ 4y - 5(10) = 2$
$\therefore\ y = 13$

g $15x - 4y = 6$
$$\begin{array}{rl} -(18x - 4y &= 10) \\ \hline -3x &= -4 \end{array}$$
$\therefore\ x = \frac{4}{3}$
$9\left(\frac{4}{3}\right) - 2y = 5$
$-2y = -7, \therefore\ y = \frac{7}{2}$

h $2p + 5q = -3$
$$\begin{array}{rl} 7p - 2q &= 9 \\ \hline \end{array}$$
$4p + 10q = -6\ 39p = 39$
$$\begin{array}{rl} 35p - 10q &= 45 \\ \hline p = 1 & \end{array}$$
$\therefore\ q = -1$

i $2x - 4y = -12$
$$\begin{array}{rl} 6x + 4y &= 4 \\ \hline 8x &= -8 \end{array}$$
$\therefore\ x = -1$
$2y - 3 - 2 = 0, \therefore\ y = \frac{5}{2}$

4 a $3x + y = 6$ (1)
$6x + 2y = 7$ (2)
Multiply (1) by 2.
$6x + 2y = 12$ (3)
Subtract (2) from (3)
$0 = 5$.
There are no solutions.
The graphs of the two straight lines are parallel.

b $3x + y = 6$ (1)
$6x + 2y = 12$ (2)
Multiply (1) by 2.
$6x + 2y = 12$ (3)
Subtract (2) from (3)
$0 = 0$.
There are infinitely many solutions.
The graphs of the two straight lines coincide.

c $3x + y = 6$ (1)
$6x - 2y = 7$ (2)
Multiply (1) by 2.
$6x + 2y = 12$ (3)
Add (2) and (3)
$12x = 19$.
$x = \frac{19}{12}$ and $y = \frac{5}{4}$. There is only one solution.

The graphs intersect at the point$\left(\frac{19}{12}, \frac{5}{4}\right)$

d $3x - y = 6$ (1)

$6x + 2y = 7$ (2)

Multiply (1) by 2.

$6x - 2y = 12$ (3)

Add (2) and (3)

$12x = 19.$

$x = \frac{19}{12}$ and $y = -\frac{5}{4}$. There is only one solution.

The graphs intersect at the point$\left(\frac{19}{12}, -\frac{5}{4}\right)$

Solutions to Exercise 1D

1 $x + y = 138$

$x - y = 88$

$2x = 226$

$\therefore x = 113$

$y = 138 - 113 = 25$

2 $x + y = 36$

$x - y = 9$

$2x = 45$

$\therefore x = 22.5$

$y = 36 - 22.5 = 13.5$

3 $6S + 4C = 58$

$5S + 2C = 35, \therefore\ 10S + 4C = 70$

$10S + 4C = 70$

$-(6S + 4C) = 58$

$4S = 12$

$\therefore\ S = \$3$

$2C = 35 - 35, \therefore\ C = \$10$

a $10S + 4C = 10 \times 3 + 4 \times 10$

$= 30 + 40 = \$70$

b $4S = 4 \times 3 = \$12$

c $S = \$3$

4 $7B + 4W = 213$

$B + W = 42, \therefore\ 4B + 4W = 168$

$7B + 4W = 213$

$-(4B + 4W = 168)$

$3B = 45$

$\therefore\ B = 15$

$15 + W = 42, \therefore\ W = \$27$

a $4B + 4W = 4 \times 15 + 4 \times 27$

$= 60 + 108 = \$168$

b $3B = 3 \times 15 = \$45$

c $B = \$15$

5 $x + y = 45$

$x - 7 = 11$

$2x = 56$

$\therefore x = 28;\ y = 17$

6 $m + 4 = 3(c + 4) \ldots (1)$

$m - 2 = 5(c - 4) \ldots (2)$

From (1), $m = 3c + 8$.

Substitute into (2):

$3c + 8 - 4 = 5(c - 4)$

$3c + 4 = 5c - 20$

$-2c = -24, \therefore\ c = 12$

$\therefore\ m - 4 = 5(12 - 4)$

$m = 44$

7 $h = 5p$

$h + p = 20$

$\therefore\ 5p + p = 30$

$\therefore\ p = 5;\ h = 25$

8 Let one child have x marbles and the other y marbles.

$$x + y = 110$$
$$\frac{x}{2} = y - 20$$
$$\therefore \quad x = 2y - 40$$
$$\therefore \; 2y - 40 + y = 110$$
$$3y = 150$$
$$\therefore \quad y = 50; \; x = 60$$

They started with 50 and 60 marbles, and finished with 30 each.

9 Let x be the number of adult tickets and y be the number of child tickets.

$x + y = 150 \qquad (1)$

$4x + 1.5y = 560 \qquad (2)$

Multiply (1) by 1.5.

$1.5x + 1.5y = 225 \qquad (1')$

Subtract (1') from (2)

$2.5x = 335$

$x = 134$

Substitute in (1). $y = 16$

There were 134 adult tickets and 16 child tickets sold.

10 Let a be the numerator and b be the denominator.

$a + b = 17 \qquad (1)$

$\frac{a+3}{b} = 1 \qquad (2).$

From (2), $a + 3 = b \qquad (1')$

Substitute in (1)

$a + a + 3 = 17$

$2a = 14$

$a = 7$ and hence $b = 10$.

The fraction is $\frac{7}{10}$

11 Let the digits be m and n.

$m + n = 8 \qquad (1)$

$10n + m - (n + 10m) = 36$

$9n - 9m = 36$

$n - m = 4 \qquad (2)$

Add (1) and (2)

$2n = 12$ implies $n = 6$.

Hence m = 2.

The initial number is 26 and the second number is 62.

12 Let x be the number of adult tickets and y be the numbr of child tickets.

$x + y = 960 \qquad (1)$ $30x + 12y = 19\,080$ (2)

Multiply (1) by 12. $12x + 12y = 11\,520$ (1')

Subtract (1') from (2).

$18x = 7560$

$x = 420.$

There were 420 adults and 540 children.

13 $0.1x + 0.07y = 1400 \ldots (1)$

$0.07x + 0.1y = 1490 \ldots (2)$

From (1), $x = (14\,000 - 0.7y)$

From (2):

$$0.07(14\,000 - 0.7y) + 0.1y = 1490$$
$$\therefore \quad 980 - 0.049\,y + 0.1y = 1490$$
$$0.051y = 510$$
$$\therefore \quad y = \frac{510}{.051}$$
$$= 10\,000$$

From (1):

$$0.1x + 0.07 \times 10\,000 = 1400$$
$$0.1x = 1400 - 700$$
$$= 700$$
$$\therefore \quad x = 7000$$

So $x + y = \$17\,000$ invested.

14 $\frac{100s}{3} + 20t = 10\,000 \ldots (1)$

$$\left(\frac{100}{3}\right)\left(\frac{s}{2}\right) + 20\left(\frac{2t}{3}\right) = 6000$$

$$\therefore \quad \left(\frac{50s}{3}\right) + \frac{40t}{3} = 6000 \ldots (2)$$

From (1):

$$20t = 10\,000 - \frac{100s}{3}$$

$$\therefore \; t = 500 - \frac{5s}{3} \ldots (3)$$

Substitute into (2):

$$\left(\frac{50s}{3}\right) + \left(\frac{40}{3}\right)\left(500 - \frac{5s}{3}\right) = 6000$$

$$150s + 120\left(500 - \frac{5s}{3}\right) = 54\,000$$

$$150s + 60\,000 - 200s = 54\,000$$

$$-50s = -6000$$

$$\therefore \quad s = 120$$

Substitute into (3):

$$t = 500 - \left(\frac{5}{3}\right) \times 120$$

$$= 500 - 200$$

$$\therefore \; t = 300$$

He sold 120 shirts and 300 ties.

15 Outback = x, BushWalker = y; $x = 1.2$y

$$200x + 350y = 177\,000$$

$$200(1.2y) + 350y = 177\,000$$

$$240y + 350y = 177\,000$$

$$\therefore \quad y = \frac{177\,000}{590} = 300$$

$$\therefore \quad x = 1.2 \times 300$$

$$= 360$$

16 Mydney = x jeans; Selbourne = y jeans

$$30x + 28\,000 = 24y + 35\,200 \ldots (1)$$

$$x + y = 6000 \ldots (2)$$

From (2): $y = 6000 - x$

Substitute in (1):

$$30x + 28\,000 = 24(6000 - x) + 35\,200$$

$$30x + 28\,000 = 144\,000 - 24x + 35\,200$$

$$54x = 151\,200$$

$$\therefore \quad x = 2800\,;\, y = 3200$$

17 Tea $A = \$10$; $B = \$11$, $C = \$12$ per kg

$B = C$; $C + B + A = 100$

$$10A + 11B + 12C = 1120$$

$$10A + 23B = 1120$$

$$\therefore \quad A = 100 - 2B$$

$$10(100 - 2B) + 23B = 1120$$

$$3B = 1120 - 1000$$

$$\therefore \quad B = 40$$

$A = 20$kg, $B = C = 40$ kg

Solutions to Exercise 1E

1 a $x + 3 < 4$

$x < 4 - 3, \therefore\ x < 1$

b $x - 5 > 8$

$x > 8 + 5, \therefore\ x > 13$

c $2x \geq 6$

$\frac{2x}{2} \geq \frac{6}{2}, \therefore\ x \geq 3$

d $\frac{x}{3} \leq 4$

$3\left(\frac{x}{3}\right) \leq 12, \therefore\ x \leq 12$

e $-x \geq 6$

$0 \geq 6 + x$

$-6 \geq x, \therefore\ x \leq -6$

f $-2x < -6$

$-x < -3$

$0 < x - 3$

$3 < x, \therefore\ x > 3$

g $6 - 2x > 10$

$3 - x > 5$

$-x > 2$

$0 > x + 2$

$-2 > x, \therefore x < -2$

h $-\frac{3x}{4} \leq 6$

$-x \leq 8$

$0 \leq x + 8$

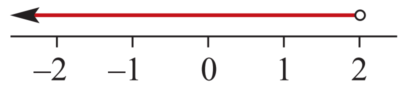
$-8 \leq x, \therefore\ x \geq -8$

i $4x - 4 \leq 2$

$x - 1 \leq \frac{1}{2}, \therefore\ x \leq \frac{3}{2}$

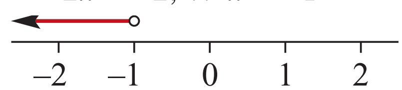
2 a $4x + 3 < 11$

$4x < 8, \therefore\ x < 2$



b $3x + 5 < x + 3$

$2x < -2, \therefore\ x < -1$

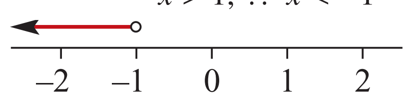


c $\frac{1}{2}(x + 1) - x > 1$

$\frac{x}{2} + \frac{1}{2} - x > 1$

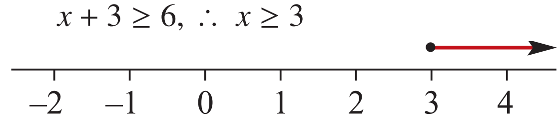
$-\frac{x}{2} > \frac{1}{2}$

$-x > 1, \therefore\ x < -1$



d $\frac{1}{6}(x + 3) \geq 1$

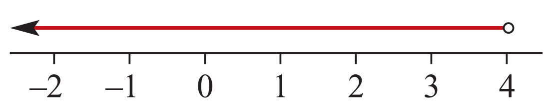
$x + 3 \geq 6, \therefore\ x \geq 3$



e $\frac{2}{3}(2x - 5) < 2$

$2x - 5 < 3$

$2x < 8, \therefore\ x < 4$

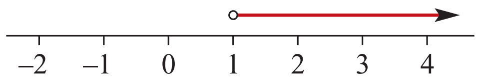


f $\dfrac{3x-1}{4} - \dfrac{2x+3}{2} < -2$

$(3x-1) - (4x+6) < -8$

$-x - 7 < -8$

$-x < -1, \therefore\ x > 1$



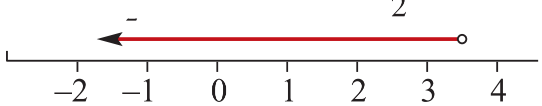
g $\dfrac{4x-3}{2} - \dfrac{3x-3}{3} < 3$

$\dfrac{4x-3}{2} - (x-1) < 3$

$4x - 3 - (2x-2) < 6$

$2x - 1 < 6$

$2x < 7, \therefore\ x < \dfrac{7}{2}$

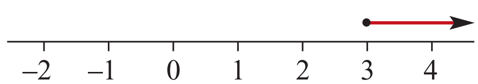


h $\dfrac{1-7x}{-2} \geq 10$

$\dfrac{7x-1}{2} \geq 10$

$7x - 1 \geq 20$

$7x \geq 21, \therefore x \geq 3$

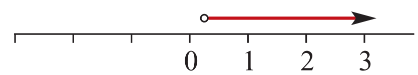


i $\dfrac{5x-2}{3} - \dfrac{2-x}{3} > -1$

$(5x-2) - (2-x) > -3$

$6x - 4 > -3$

$6x > 1, \therefore x > \dfrac{1}{6}$



3 **a** $2x + 1 > 0$

$2x > -1, \therefore x > -\dfrac{1}{2}$

b $100 - 50x > 0$

$100 > 50x$

$2 > x, \therefore x < 2$

c $100 + 20x > 0$

$20x > -100, \therefore x > -5$

4 Let p be the number of sheets of paper.

$3p < 20$

$p < \dfrac{20}{3}$

$p \in \mathbb{Z}, \therefore\ p = 6$

5 $\dfrac{66 + 72 + x}{3} \geq 75$

$138 + x \geq 225$

$\therefore\ x \geq 87$

Lowest mark:87

Solutions to Exercise 1F

1 **a** $c = ab$

$= 6 \times 3 = 18$

b $r = p + q$

$= 12 + -3 = 9$

c $c = ab$

$\therefore\ b = \dfrac{c}{a}$

$= \dfrac{18}{6} = 3$

d $r = p + q$

$\therefore\ q = r - p$

$= -15 - 3 = -18$

e $c = \sqrt{a}$

$= \sqrt{9} = 3$

f $c = \sqrt{a}$

$\therefore\ a = c^2$

$= 9^2 = 81$

g $p = \dfrac{u}{v}$

$= \dfrac{10}{2} = 5$

h $p = \dfrac{u}{v}$

$\therefore\ u = pv$

$= 2 \times 10 = 20$

2 **a** $S = a + b + c$

b $P = xy$

c $C = 5p$

d $T = dp + cq$

e $T = 60a + b$

3 **a** $E = IR$

$= 5 \times 3 = 15$

b $C = pd$

$= 3.14 \times 10 = 31.4$

c $P = R\left(\dfrac{T}{V}\right)$

$= 60 \times \dfrac{150}{9} = 1000$

d $I = \dfrac{E}{R}$

$= \dfrac{240}{20} = 12$

e $A = \pi rl$

$= 3.14 \times 5 \times 20 = 314$

f $S = 90(2n - 4)$

$= 90(6 \times 2 - 4) = 720$

4 **a** $PV = c, \therefore\ V = \dfrac{c}{P}$

b $F = ma, \therefore\ a = \dfrac{F}{m}$

c $I = Prt, \therefore\ P = \dfrac{I}{rt}$

d $w = H + Cr$

$\therefore Cr = w - H$

$\therefore\ r = \dfrac{w - H}{C}$

e $S = P(1 + rt)$

$\therefore \dfrac{S}{P} = 1 + rt$

$\therefore rt = \dfrac{S}{P} - 1 = \dfrac{S - P}{P}$

$\therefore t = \dfrac{S - P}{rP}$

f $V = \dfrac{2R}{R - r}$

$\therefore \ (R - r)V = 2R$

$V - rV - 2R = 0$

$R(V - 2) = rV$

$\therefore \quad r = \dfrac{R(V - 2)}{V}$

5 a $D = \dfrac{T + 2}{P}$

$10 = \dfrac{T + 2}{5}$

$T + 2 = 50, \therefore \ T = 48$

b $A = \dfrac{1}{2}bh$

$40 = \dfrac{10b}{2}$

$10b = 80, \therefore \ b = 8$

c $V = \dfrac{1}{3}\pi hr^2$

$\therefore h = \dfrac{3V}{\pi r^2}$

$= \dfrac{300}{25 \times 3.14}$

$= \dfrac{12}{3.14} = 3.82$

d $A = \dfrac{1}{2}h(a + b)$

$50 = \dfrac{5}{2} \times (10 + b)$

$20 = 10 + b, \therefore \ b = 10$

6 a $l = 4a + 3w$

b $H = 2b + h$

c $A = 3 \times (h \times w) = 3hw$

d

$Area = H \times l - 3hw$

$= (4a + 3w)(2b + h) - 3hw$

$= 8ab + 6bw + 4ah + 3hw - 3hw$

$= 8ab + 6bw + 4ah$

7 a i Circle circumferences $= 2\pi(p + q)$
Total wire length
$T = 2\pi(p + q) + 4h$

ii $\mathrm{T} = 2\pi(20 + 24) + 4 \times 28$

$= 88\pi + 112$

b $A = \pi h(p + q)$

$\therefore \ p + q = \dfrac{A}{\pi h}$

$\therefore p = \dfrac{A}{\pi h} - q$

8 a $P = \dfrac{T - M}{D}$

$6 = \dfrac{8 - 4}{D}$

$6D = 4, \therefore \ D = \dfrac{2}{3}$

b $H = \dfrac{a}{3} + \dfrac{a}{b}$

$5 = \dfrac{6}{3} + \dfrac{6}{b}$

$\dfrac{6}{b} = 5 - 2 = 3$

$3b = 6, \therefore \ b = 2$

c $a = \dfrac{90(2n-4)}{n}$

$6 = \dfrac{90(2n-4)}{n}$

$6n = 90(2n-4)$

$n = 15(2n-4)$

$n = 30n - 60$

$29n = 30, \therefore\ n = \dfrac{60}{29}$

d $R = \dfrac{r}{a} + \dfrac{r}{3}$

$4 = \dfrac{r}{2} + \dfrac{r}{3}$

$\dfrac{5r}{6} = 4$

$\therefore\ r = \dfrac{24}{5} = 4.8$

9 a Big triangle area $= \dfrac{1}{2}bc$

Small triangle area $= \dfrac{1}{2}bk \times ck$

$= \dfrac{1}{2}bck^2$

Shaded area $D = \dfrac{1}{2}bc(1-k^2)$

b $D = \dfrac{1}{2}bc(1-k^2)$

$1 - k^2 = \dfrac{2D}{bc}$

$k^2 = 1 - \dfrac{2D}{bc}$

$\therefore\ k = \sqrt{1 - \dfrac{2D}{bc}}$

c $k = \sqrt{1 - \dfrac{2D}{bc}}$

$= \sqrt{1 - \dfrac{4}{12}}$

$= \sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{6}}{3}$

10 a Width of each arm $= c$

Length of each ofthe 8 arms $= \dfrac{b-c}{2}$

$P = 8 \times \dfrac{b-c}{2} + 4c$

$= 4b - 4c + 4c = 4b$

b Area of each piece $= bc$, but the centre area (c^2) is counted twice

$\therefore\ A = 2bc - c^2$

c $2bc = A + c^2$

$\therefore\ b = \dfrac{A + c^2}{2c}$

11 a $a = \sqrt{a + 2b}$

$a^2 = a + 2b$

$2b = a(a-1)$

$\therefore\ b = \dfrac{a}{2}(a-1)$

b

$\dfrac{a+x}{a-x} = \dfrac{b-y}{b+y}$

$(a+x)(b+y) = (a-x)(b-y)$

$ab + bx + ay + xy = ab - bx - ay + xy$

$bx + ay = -bx - ay$

$2bx + 2ay = 0$

$2bx = -2ay$

$\therefore\ x = -\dfrac{ay}{b}$

c $px = \sqrt{3q - r^2}$

$p^2x^2 = 3q - r^2$

$r^2 = 3q - p^2x^2$

$\therefore\ r = \pm\sqrt{3q - p^2x^2}$

d $$\frac{x}{y} = \sqrt{1 - \frac{v^2}{u^2}}$$

$$\frac{x^2}{y^2} = 1 - \frac{v^2}{u^2}$$

$$\frac{v^2}{u^2} = 1 - \frac{x^2}{y^2} = \frac{y^2 - x^2}{y^2}$$

$$v^2 = \frac{u^2}{y^2}(y^2 - x^2)$$

$$\therefore \ v = \pm\frac{u}{y}\sqrt{y^2 - x^2}$$

$$= \pm\sqrt{(u^2)\left(1 - \frac{x^2}{y^2}\right)}$$

Solutions to Review: Short-answer questions

1 a $2x + 6 = 8$

$2x = 2, \therefore x = 1$

b $3 - 2x = 6$

$-2x = 3, \therefore x = -\frac{3}{2}$

c $2x + 5 = 3 - x$

$\therefore \quad 3x = -2, \therefore x = -\frac{2}{3}$

d $\frac{3 - x}{5} = 6$

$3 - x = 30$

$-x = 27, \therefore x = -27$

e $\frac{x}{3} = 4, \therefore x = 12$

f $\frac{13x}{4} - 1 = 10$

$\frac{13x}{4} = 11$

$13x = 44, \therefore \ x = \frac{44}{13}$

g $3(2x + 1) = 5(1 - 2x)$

$6x + 3 = 5 - 10x$

$16x = 2, \therefore x = \frac{1}{8}$

h $\frac{3x + 2}{5} + \frac{3 - x}{2} = 5$

$2(3x + 2) + 5(3 - x) = 50$

$6x + 4 + 15 - 5x = 50$

$\therefore \quad x = 50 - 19 = 31$

2 a $a - t = b$

$a = t + b, \therefore t = a - b$

b $\frac{at + b}{c} = d$

$at + b = cd$

$at = cd - b$

$\therefore \quad t = \frac{cd - b}{a}$

c $a(t - c) = d$

$at - ac = d$

$at = d + ac$

$\therefore \quad t = \frac{d + ac}{a} = \frac{d}{a} + c$

d $\frac{a - t}{b - t} = c$

$a - t = c(b - t)$

$a - t = cb - ct$

$-t + ct = cb - a$

$t(c - 1) = cb - a$

$\therefore \quad t = \frac{cb - a}{c - 1}$

e $\frac{at + b}{ct - b} = 1$

$at + b = ct - b$

$at - ct = -2b$

$t(c - a) = 2b$

$\therefore \quad t = \frac{2b}{c - a}$

f $\dfrac{1}{at+c} = d$

$dat + dc = 1$

$dat = 1 - dc$

$\therefore \quad t = \dfrac{1-dc}{ad}$

3 **a** $2 - 3x > 0$

$2 > 3x$

$\dfrac{2}{3} > x, \therefore x < \dfrac{2}{3}$

b $\dfrac{3-2x}{5} \geq 60$

$3 - 2x \geq 300$

$-2x \geq 297$

$-297 \geq 2x$

$-\dfrac{297}{2} \geq x$

$\therefore x \leq -148.5$

c $3(58x - 24) + 10 < 70$

$3(58\ x - 24) < 60$

$58x - 24 < 20$

$58x < 44, \ \therefore \ x < \dfrac{22}{29}$

d $\dfrac{3-2x}{5} - \dfrac{x-7}{6} \leq 2$

$6(3\ - 2x) - 5(x - 7) \leq 60$

$18 - 12x - 5x - 35 \leq 60$

$53 - 17x \leq 60$

$-17x \leq 7$

$0 \leq \ 17x + 7$

$-\dfrac{7}{17} \leq x$

$\therefore \quad x \geq \ -\dfrac{7}{17}$

4 $z = \dfrac{x}{2} - 3t$

$\dfrac{1}{2}x = z + 3t$

$\therefore \quad x = 2z + 6t$

When $z = 4$ and $t = -3$:

$x = 2 \times 4 + 6 \times -3$

$= 8 - 18 = -10$

5 **a** $d = e^2 + 2f$

b $d - e^2 = 2f$

$\therefore \quad f = \dfrac{1}{2}(d - e^2)$

c If $d = 10$ and $e = 3$,

$f = \dfrac{1}{2}(10 - 3^2) = \dfrac{1}{2}$

6 $A = 400\pi \text{ cm}^3$

7 The volume of metal in a tube is given by the formula $V = \pi\ell[r^2 - (r - t)^2]$, where ℓ, is the length of the tube, r is the radius of the outside surface and t is the thickness of the material.

a $\ell = 100, r = 5$ and $t = 0.2$

$V = \pi \times 100[5^2 - (5 - 0.2)^2]$

$= \pi \times 100(5 - 4.8)(5 + 4.8)$

$= \pi \times 100 \times 0.2 \times 9.8$

$= \pi \times 20 \times 9.8$

$= 196\pi$

b $\ell = 50, r = 10$ and $t = 0.5$

$$V = \pi \times 50[10^2 - (10 - 0.5)^2]$$
$$= \pi \times 50(10 - 9.5)(10 + 9.5)$$
$$= \pi \times 50 \times 0.5 \times 19.5$$
$$= \pi \times 25 \times 19.5$$
$$= \frac{975\pi}{2}$$

8 **a** $A = \pi rs \qquad (r)$

$$A = \pi rs$$
$$r = \frac{A}{\pi s}$$

b $T = P(1 + rw) \qquad (w)$

$$T = P(1 + rw)$$
$$T = P + Prw$$
$$T - P = Prw$$
$$w = \frac{T - P}{Pr}$$

c $v = \sqrt{\frac{n - p}{r}} \qquad (r)$

$$v^2 = \frac{n - p}{r}$$
$$r \times v^2 = n - p$$
$$r = \frac{n - p}{v^2}$$

d $ac = b^2 + bx \qquad (x)$

$$ac = b^2 + bx$$
$$ac - b^2 = bx$$
$$x = \frac{ac - b^2}{b}$$

9 $s = \left(\frac{u + v}{2}\right)t.$

a $u = 10, v = 20$ and $t = 5.$

$$s = \left(\frac{10 + 20}{2}\right) \times 5$$
$$= 75$$

b $u = 10, v = 20$ and $s = 120.$

$$120 = \left(\frac{10 + 20}{2}\right)t$$
$$120 = 15t$$
$$t = 8$$

10 $V = \pi r^2 h$ where r cm is the radius and h cm is the height
$V = 500\pi$ and $h = 10.$

$$500\pi = \pi r^2 \times 10$$

$r^2 = 50$ and therefore $r = 5\sqrt{2}$
The radius is $r = 5\sqrt{2}$ cm.

11 Let the lengths be x m and y m.
$10x + 5y = 205 \qquad (1)$
$3x - 2y = 2 \qquad (2)$
Multiply (1) by 2 and (2) by 5.
$20x + 10y = 410 \qquad (3)$
$15x - 10y = 10 \qquad (4)$
Add (3) and (4)
$35x = 420$
$x = 12$ and $y = 17.$
The lengths are 12 m and 17 m.

12 $\frac{m + 1}{n} = \frac{1}{5} \qquad (1).$
$\frac{m}{n - 1} = \frac{1}{7} \qquad (2).$
They become:
$5m + 5 = n \qquad (1)$ and $7m = n - 1$
(2)
Substitute from (1) in (2).
$7m = 5m + 5 - 1$

$m = 2$ and $n = 15$.

13 ■ Mr Adonis earns \$7200 more than Mr Apollo

■ Ms Aphrodite earns \$4000 less than Mr Apollo.

■ If the total of the three incomes is \$303 200, find the income of each person.

Let Mr Apollo earn $\$x$.
Mr Adonis earns $\$(x + 7200)$
Ms Aphrodite earns $\$(x - 4000)$
We have

$$\begin{aligned} x + x + 7200 + x - 4000 &= 303\,200 \\ 3x + 3200 &= 303\,200 \\ 3x &= 300\,000 \\ x &= 100\,000 \end{aligned}$$

Mr Apollo earns \$100 000 ; Mr Adonis earns \$107 200 and Ms Aphrodite earns \$96 000.

14 **a** $4a - b = 11$ (1)
$3a + 2b = 6$ (2)
Multiply (1) by 2.
$8a - 2b = 22$ (3)
Add (3) and (2).
$11a = 28$ which implies $a = \frac{28}{11}$.
From(1), $b = -\frac{9}{11}$

b $a = 2b + 11$ (1)
$4a - 3b = 11$ (2)
Substitute from (1) in (2).
$4(2b + 11) - 3b = 11$
$5b = -33$
$b = -\frac{33}{5}$
From (1), $a = 2 \times \left(-\frac{33}{5}\right) + 11 = -\frac{11}{5}$.

15 Let t_1 hours be the time spent on highways and t_2 hours be the time travelling through towns.
$t_1 + t_2 = 6$ (1)
$80t_1 + 24t_2 = 424$ (2)
From (1) $t_2 = 6 - t_1$
Substitute in (2).
$80t_1 + 24(6 - t_1) = 424$
$56t_1 = 424 - 6 \times 24$
$t_1 = 5$ and $t_2 = 1$.
The car travelled for 5 hours on highways and 1 hour through towns.

Solutions to Review: Multiple-choice questions

1 D $3x - 7 = 11$

$3x = 18$

$x = 6$

2 D $\frac{x}{3} + \frac{1}{3} = 2$

$x + 1 = 6$

$x = 5$

3 C $x - 8 = 3x - 16$

$-2x = -8$

$x = 4$

4 A $7 = 11(x - 2)$

5 C $2(2x - y) = 10$

$\therefore\ 4x - 2y = 20$

$x + 2y = 0$

$5x = 20$

$\therefore\ x = 4; y = -2$

6 C Average cost = total $/total items

$= \frac{ax + by}{x + y}$

7 B $\frac{x+1}{4} - \frac{2x-1}{6} = x$

$3(x + 1) - 2(2x - 1) = 12x$

$3x + 3 - 4x + 2 = 12x$

$-13x = -5$

$\therefore\ x = \frac{5}{13}$

8 B $\frac{72 + 15z}{3} > 4$

$72 + 15z > 12$

$15z > -60$

$\therefore\ z > -4$

9 A $A = \frac{hw + k}{w}$

$Aw = hw + k$

$w(A - h) = k$

$\therefore\ w = \frac{k}{A - h}$

10 B Total time taken (hrs)

$= \frac{x}{2.5} + \frac{8x}{5} = \frac{1}{2}$

$\frac{2x}{5} + \frac{8x}{5} = \frac{1}{2}$

$\frac{10x}{5} = \frac{1}{2},\ \therefore\ x = \frac{1}{4}$

$x = \frac{1}{4}$ km = 250 m

11 E The lines $y = 2x + 4$ and $y = 2x + 6$ are parallel but have different y-axis intercepts.
Alternatively if $2x + 4 = 2x + 6$ then $4 = 6$ which is impossible.

12 B $5(x + 3) = 5x + 15$ for all x.

Solutions to Review: Extended-response questions

1 a $F = \frac{9}{5}C + 32$

If $F = 30$, then $30 = \frac{9}{5}C + 32$

and $\frac{9}{5}C = -2$

which implies $C = -\frac{10}{9}$

A temperature of 30°F corresponds to $\left(-\frac{10}{9}\right)^\circ$C.

b If $C = 30$, then $F = \frac{9}{5} \times 30 + 32$

$= 54 + 32 = 86$

A temperature of 30°C corresponds to a temperature of 86°F.

c x°C = x° F when $x = \frac{9}{5}x + 32$

$-\frac{4}{5}x = 32$

$\therefore \quad x = -40$

Hence -40°F = -40°C.

d $x = \frac{9}{5}(x + 10) + 32$

$5x = 9x + 90 + 160$

$-4x = 250$

$\therefore \quad x = -62.5$

e $x = \frac{9}{5}(2x) + 32$

$\frac{-13x}{5} = 32$

$\therefore \quad x = \frac{-160}{13}$

f $k = \frac{9}{5}(-3k) + 32$

$5k = -27k + 160$

$32k = 160$

$\therefore \quad k = 5$

2 a $$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$$

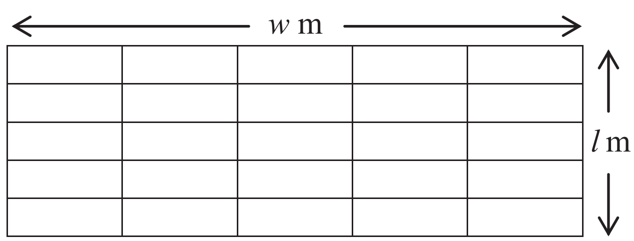
Obtain the common denominator $$\frac{u+v}{vu}=\frac{2}{r}$$

Take the reciprocal of both sides $$\frac{vu}{u+v}=\frac{r}{2}$$

Make r the subject $$r=\frac{2vu}{u+v}$$

b $$\begin{aligned} m &= \left(v-\frac{2vu}{u+v}\right)\div\left(\frac{2vu}{u+v}-u\right)\\ &= \frac{v^2-vu}{u+v}\div\frac{uv-u^2}{u+v}\\ &= \frac{v^2-vu}{u+v}\times\frac{u+v}{uv-u^2}\\ &= \frac{v(v-u)}{u(v-u)}=\frac{v}{u}\end{aligned}$$

3 a



The total length of wire is given by $T = 6w + 6l$.

b i If $w = 3l$, then $$\begin{aligned} T &= 6w+6\left(\frac{w}{3}\right)\\ &= 8w\end{aligned}$$

ii If $T = 100$, then $$8w = 100$$

Hence $$\begin{aligned} w &= \frac{25}{2}\\ l &= \frac{w}{3}\\ &= \frac{25}{6}\end{aligned}$$

c i $$L = 6x + 8y$$

Make y the subject $$8y = L - 6x$$

and $$y=\frac{L-6x}{8}$$

ii When $L = 200$ and $x = 4$,

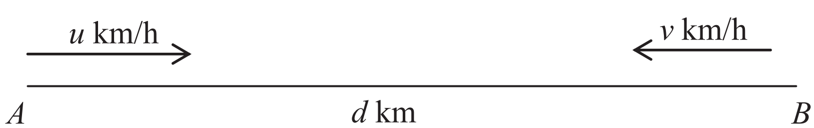
$$y = \frac{200 - 6 \times 4}{8}$$

$$= \frac{176}{8} = 22$$

d The two types of mesh give

	$6x + 8y = 100$	(1)
and	$3x + 2y = 40$	(2)
Multiply (2) by 2	$6x + 4y = 80$	(3)
Subtract (3) from (1) to give	$4y = 20$	
Hence	$y = 5$	
Substitute in (1)	$6x + 40 = 100$	
Hence	$x = 10$	

4



a At time t hours, Tom has travelled ut km and Julie has travelled vt km.

b **i** The sum ofthe two distances must be d when they meet.

Therefore $ut + vt = d$

and $t = \dfrac{d}{u + v}$

They meet after $\dfrac{d}{u + v}$ hours.

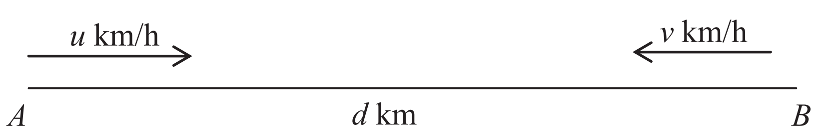
ii The distance from A is $u \times \dfrac{d}{u + v} = \dfrac{ud}{u + v}$ km.

c If $u = 30,\ v = 50$ and $d = 100$, the distance from $A = \dfrac{30 \times 100}{30 + 50}$

$= 37.5$ km

The time it takes to meet is $\dfrac{100}{30 + 50} = 1.25$ hours.

5



a The time taken to go from A to B is $\frac{d}{u}$ hours. The time taken to go from B to A is $\frac{d}{v}$ hours.

$$\text{The total time taken } = \frac{d}{u} + \frac{d}{v}$$

Therefore,

$$\begin{aligned}\text{average speed } &= 2d \div \left(\frac{d}{u} + \frac{d}{v}\right)\\ &= 2d \div \frac{dv + du}{uv}\\ &= 2d \times \frac{uv}{d(u+v)}\\ &= \frac{2uv}{u+v} \text{ km/h}\end{aligned}$$

b **i** The time to go from A to B is T hours.

Therefore $T = \frac{d}{u}$ (1)

The time for the return trip $= \frac{d}{v}$ (2)

From (1) $d = uT$

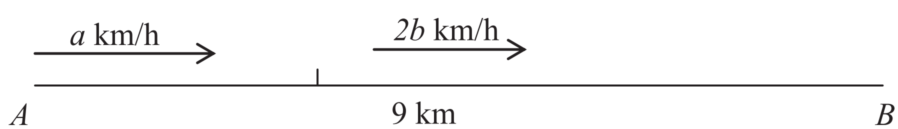
and substituting in (2) gives

the time for the return trip $= \frac{uT}{v}$.

ii The time for the entire trip $= T + \frac{uT}{v}$

$$= \frac{vT + uT}{v} \text{ hours.}$$

6 **a**



One-third of the way is 3 km.

$$\begin{aligned}\text{The time taken } &= \frac{3}{a} + \frac{6}{2b}\\ &= \frac{3}{a} + \frac{3}{b}\end{aligned}$$

b The return journey is 18 km and therefore, if the man is riding at 3*c* km/h,

$$\text{the time taken} = \frac{18}{3c} = \frac{6}{c}$$

Therefore, if the time taken to go from A to *B* at the initial speeds is equal to the time taken for the return trip travelling at 3*c* km/h,

then $\quad \frac{6}{c} = \frac{3}{a} + \frac{3}{b}$

and hence $\quad \frac{2}{c} = \frac{1}{a} + \frac{1}{b}$

c **i**

$$\frac{2}{c} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

To make *c* the subject, take the reciprocal of both sides.

$$\frac{c}{2} = \frac{ab}{a+b}$$

and $\quad c = \frac{2ab}{a+b}$

ii If $a = 10$ and $b = 20$, $c = 400 \div 30 = \frac{40}{3}$

7 **a** $\frac{x}{8}$ hours at 8 km/h

$\frac{y}{10}$ hours at 10 km/h

b

$$\begin{aligned}\text{Average speed} &= (x+y) \div \left(\frac{x}{8} + \frac{y}{10}\right)\\ &= (x+y) \div \frac{10x+8y}{80}\\ &= (x+y) \times \frac{80}{10x+8y}\\ &= \frac{80(x+y)}{10x+8y}\end{aligned}$$

c

$$10 \times \frac{x}{8} + 8 \times \frac{y}{10} = 72$$

and, from the statement of the problem,

$$x + y = 70 \qquad (1)$$

Therefore simultaneous equations in x and y

$$\frac{5x}{4} + \frac{4y}{5} = 72 \qquad (2)$$

Multiply (2) by 20 $\quad 25x + 16y = 1440 \qquad (3)$

Multiply (1) by 16 $\quad 16x + 16y = 1120 \qquad (4)$

Subtract (4) from (3)

$$9x = 320$$

which gives $\quad x = \frac{320}{9}$ and $y = \frac{310}{9}$.

8 First solve the simultaneous equations:

$2y - x = 2 \qquad (1)$

$y + x = 7. \qquad (2)$

Add (1) and (2).

$3y = 9$

$y = 3$ and from (2) $x = 4$.

Now check in

$y - 2x = -5 \qquad (3)$

LHS $= 3 - 8 = -5 =$ RHS.

The three lines intersect at $(4, 3)$.